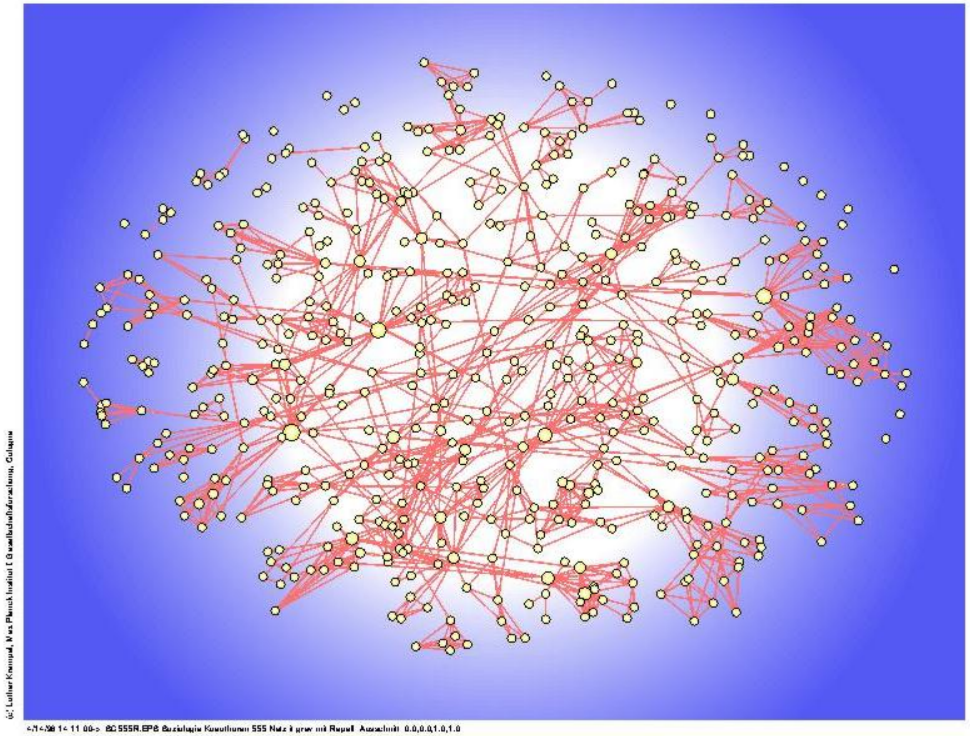

A social network: 555 scientists and their co-authorships

# Clustering of Local Optima in Combinatorial Fitness Landscapes

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Unil

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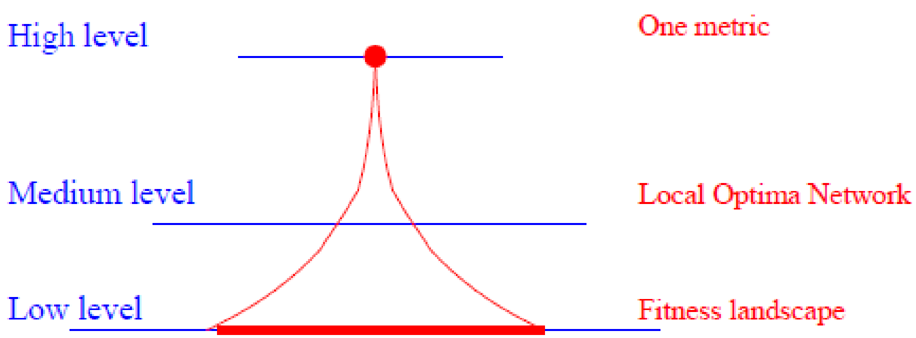
# Motivating remarks

*"Perhaps the most conspicuous limitation of a heuristic method for problems involving discrete alternatives is the ability to become trapped at a local optimum"*
Fred Glover (1986), Future Paths for Integer Programming and Links to Artificial Intelligence. *Computers and Operations Research*, 13 (5)

*"The more we know of the statistical properties of a class of fitness landscapes, the better equipped will we be for the design of effective search algorithms for such landscapes"*
L. Barnett, U. Sussex, DPhil Diss. 2003

# Overview and aims

- Use the tools of *complex networks* analysis for studying the structure of combinatorial fitness landscapes
- How? Mapping combinatorial landscapes to networks (LON) and conduct a network analysis
- Ultimate Goal: Relate (and exploit?) network features to search heuristics

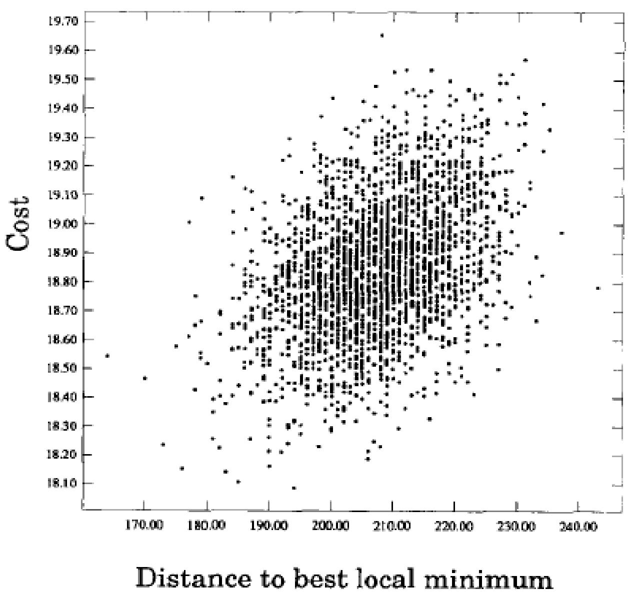


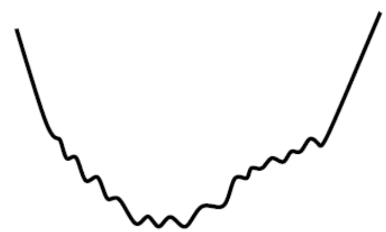

**This paper**: Use LON to study the distribution of local optima in two classes of instances of the quadratic assignment problem (QAP)

- Are the LO uniformly distributed, or
- Do they cluster in some non-homogeneous way?

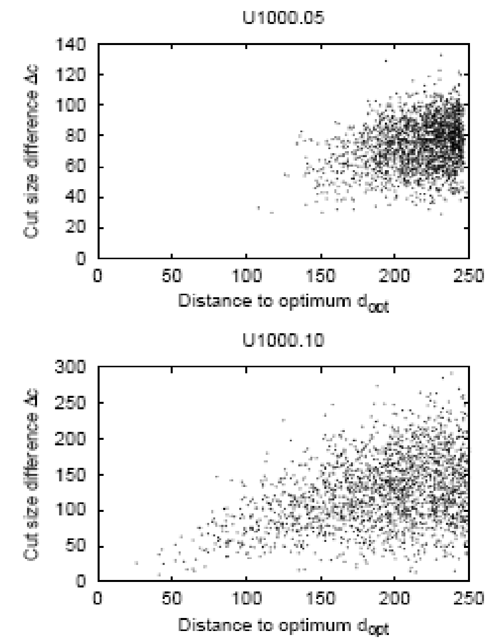
# Preliminaries: The *central-massif* or *big-valley* structure in combinatorial optimisation

- Several studies: NK landscapes (Kauffman, 1993), TSP (Boese et al, 1994), graph bipartitioning (Merz & Freisleben, 1998) flowshop scheduling (Reeves, 1999)
- Distribution of local optima is not uniform. Clustered in a big-valley (*globally convex* structure)



TSP: big-valley. Local optima confined to a small region




Graph bipartitioning: big-valley. Local minima less confined

# Preliminaries: Complex networks

## *Small-world networks*
(Watts & Strogatz, *Nature*, 1998),

**Cited by 11045**

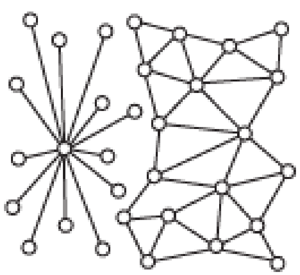
- Neither ordered nor completely random
- Nodes are highly clustered yet path length between them is small

## *Scale-free networks*
(Barabasi & Albert, *Nature*, 1999)

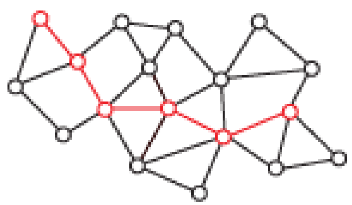
**Cited by 8073**

- Most nodes have less-than-average degree, a small fraction of hubs have a large number of connections
- Power-law degree distribution

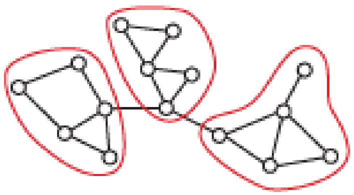
## Some features of network



**Topology**: Degree distribution



**Distance**: shortest path



**Clustering**: clustering coefficient, communities detection

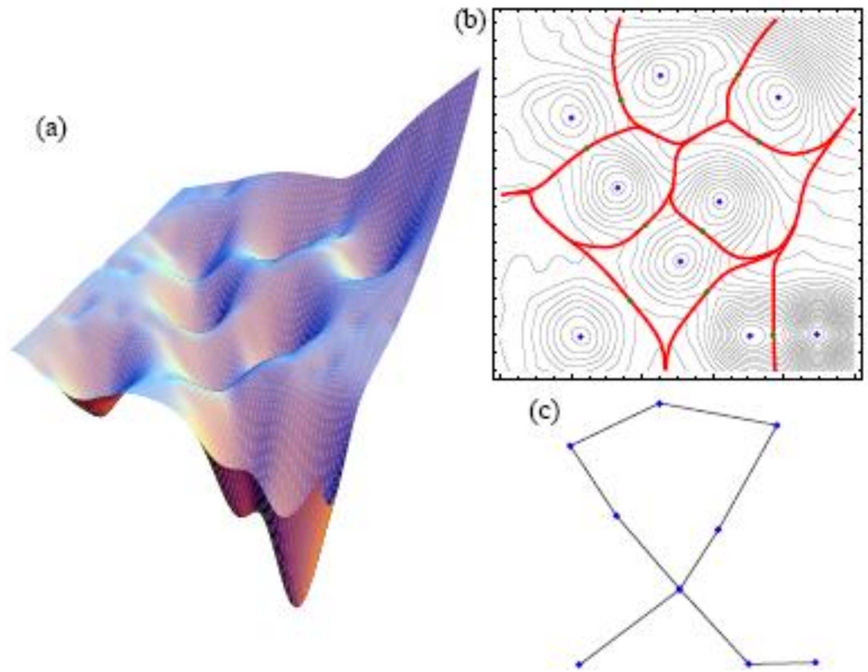
# Network view of energy surfaces

*Inherent network of energy surfaces*

- Vertices: energy minima
- Edges: two nodes are connected if energy barrier separating them is sufficiently low (transition state)

*Can we do the same for combinatorial landscapes?*

- Nodes: local optima
- Edges: transition probabilities between neighbouring basins of attraction.




(a) Model of 2D energy surface

(b) Partition of the space into basins of attraction

(c) Landscape as a network

J. P. K. Doye, The network topology of a potential energy landscape: a static scale-free network, *Phys. Rev. Lett.*, 88, 2002

# Definitions

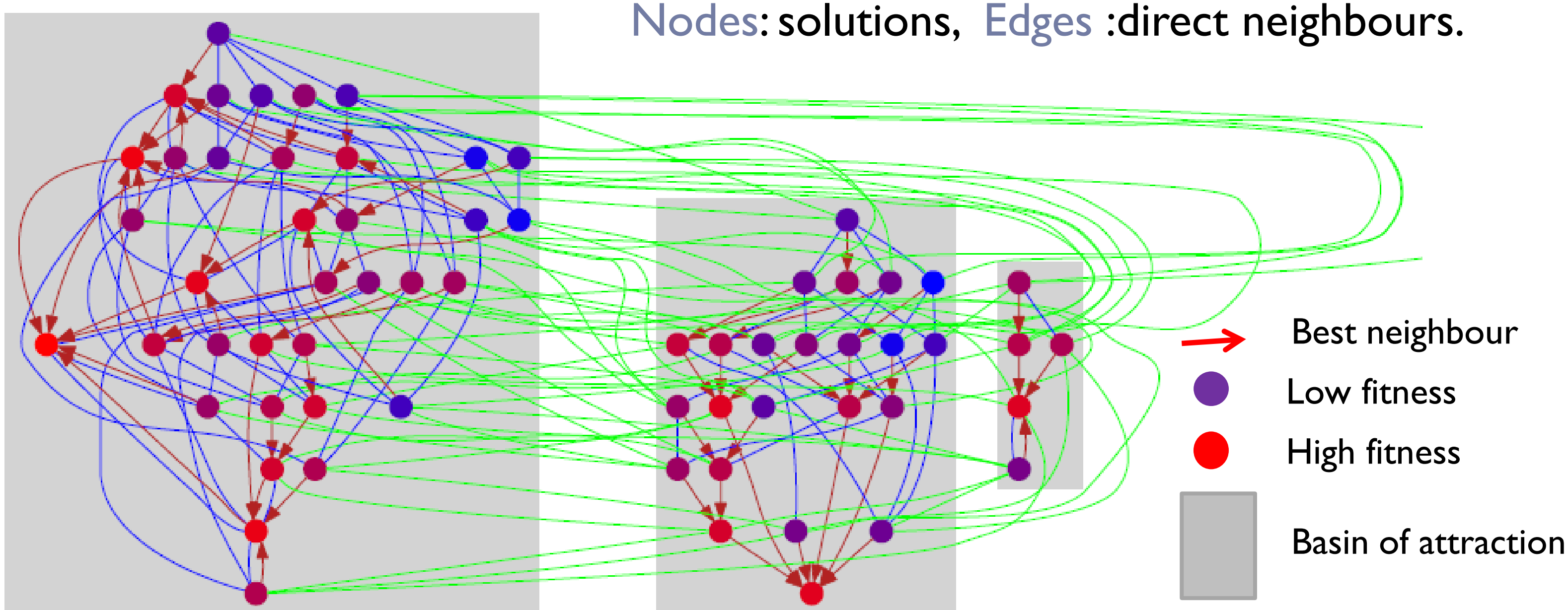
- **Local optimum**: s*, for all s in *V(s)*, *f(s)* ≤ *f(s*)*
- ***h(s)****:* associates to ***s***, the solution obtained after applying a hill-climbing algorithms (best or first) to arrive at a LO.
- Let $p_i(s)$ be the probability $P(h(s) = LO_i)$
- **Basin of attraction:** of $LO_i$, $b_i = \{s \in S \mid p_i(s) > 0\}$.
- **Edge weight** between basins $b_i$ and $b_j$: $p(b_i \rightarrow b_j)$

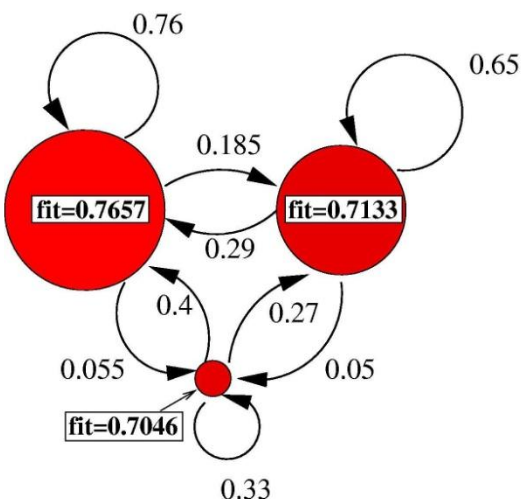
$$p(s \rightarrow b_j) = \sum_{s' \in b_j} p(s \rightarrow s')p_j(s'), \qquad p(b_i \rightarrow b_j) = \frac{1}{\sharp b_i} \sum_{s \in b_i} p_i(s)p(s \rightarrow b_j)$$

**Local optima network (LON).** ***Gw* = *(N,E)*** is the graph where nodes are the *LO*, there is an edge $e_{ij} \in E$, with weight $W_{ij} = p(b_i \rightarrow b_j)$, between *i* and *j* if $p(b_i \rightarrow b_j) > 0$.

*N*=6, *K*=2, the whole search space ($2^6$=64 solutions)

Nodes: solutions, Edges :direct neighbours.



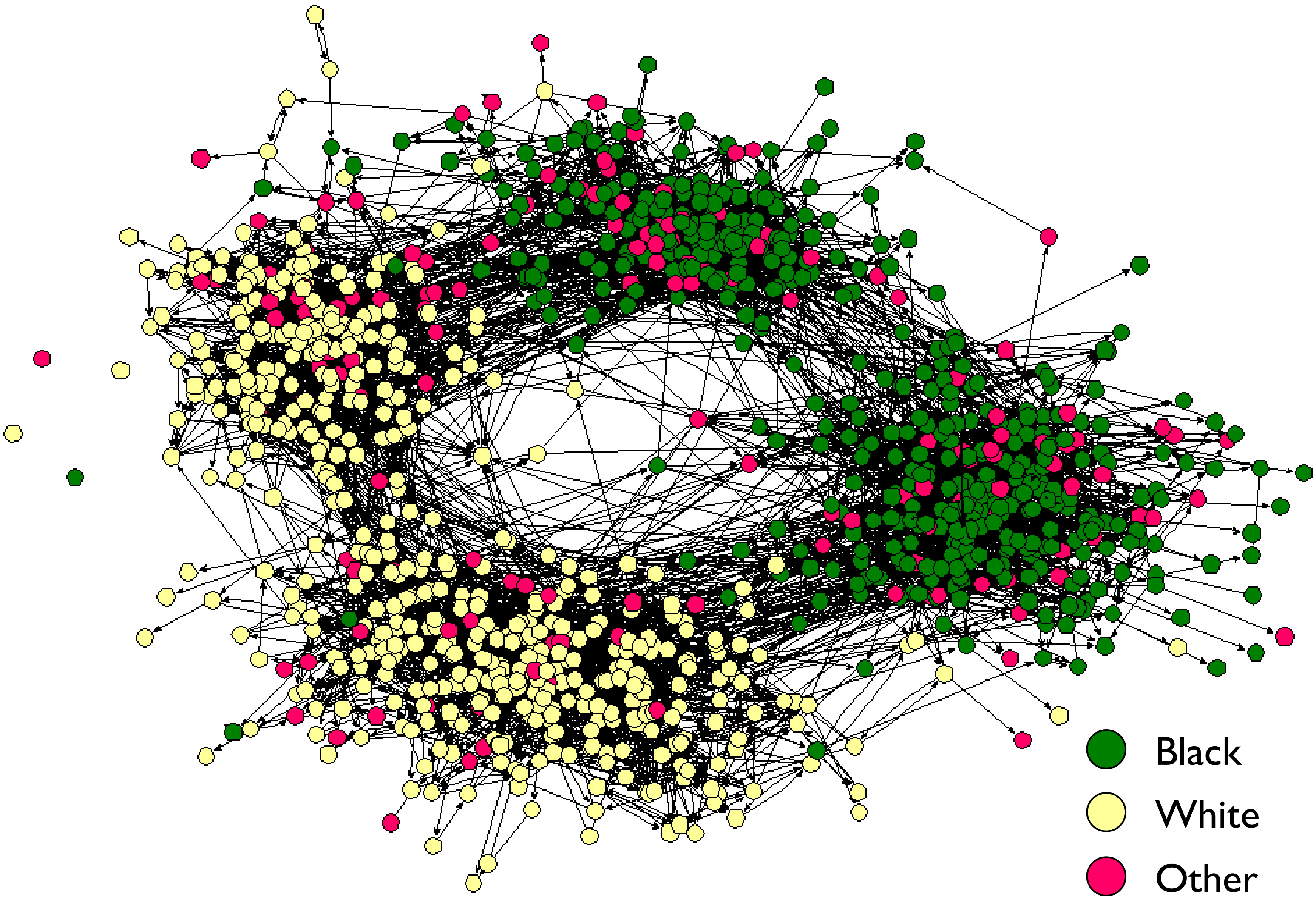




# Reduction of the landscape as a LON

Nodes: local optima, (diameter=size of basins)

Edges: transition probabilities

A network is said to have **community structure** if it divides naturally into groups of nodes with dense connections within groups and sparser connections between groups.

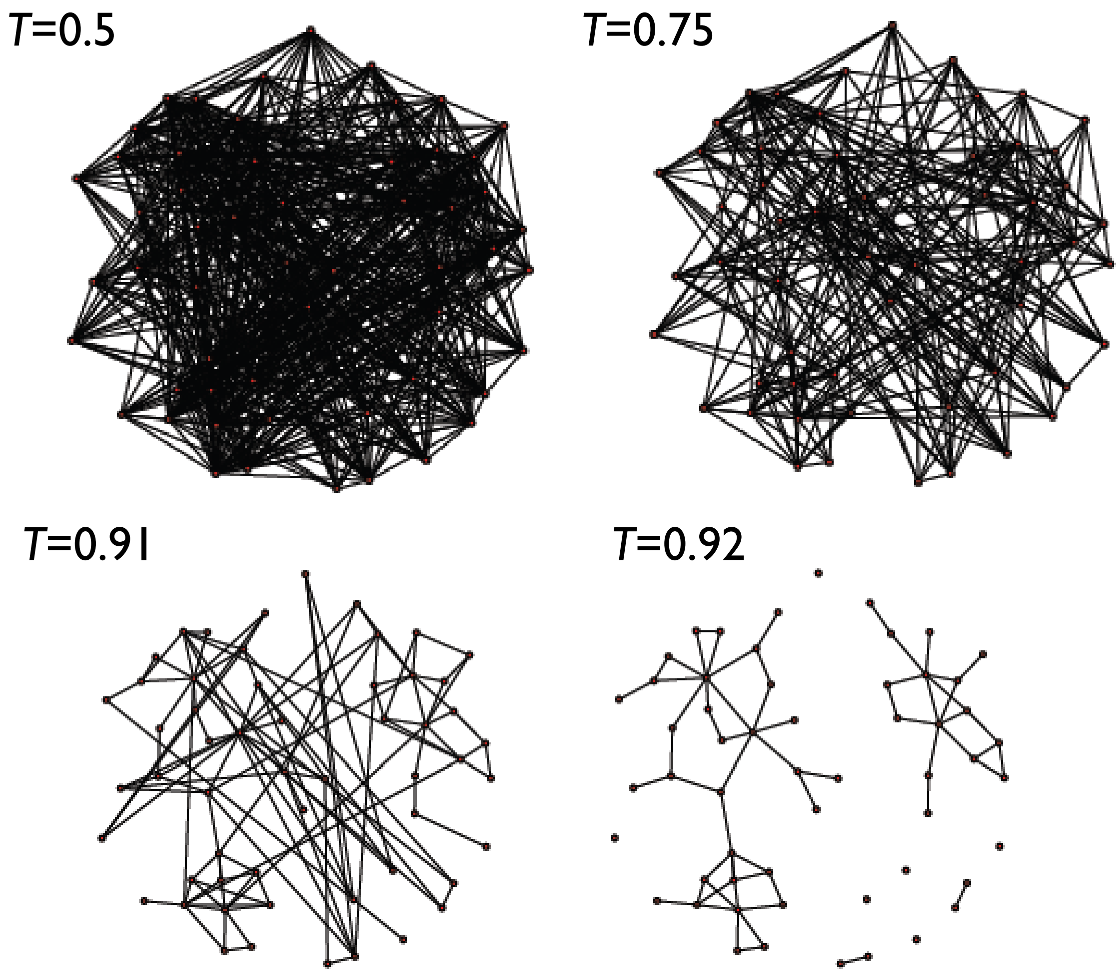



Friendship network of children in a US School.

# Methods: statistical analysis

- QAP hard combinatorial problem in permutation space: *n* units to be assigned to *n* locations (*n*x*n* flow and distance matrices)
- Pair-wise exchange operator (swap)
- Two type of instances (generators (Knowles, Corne, 2003))
  - Uniformly *random* instances:  Flows are integers sampled from uniform distributions
  - *Real-like* instances: Flow entries are non-uniform random vales, resemble the structure of QAP problems in practice
- 200 instances, *n*=9 *random* instances,  *n*=11 *real-like* instances
- *Filtering* of edges was required
- Measured community structure (*modularity* score *Q*)
  1. Greedy modularity optimization
  2. Spin glass search algorithm

# Methods: Edges Filtering

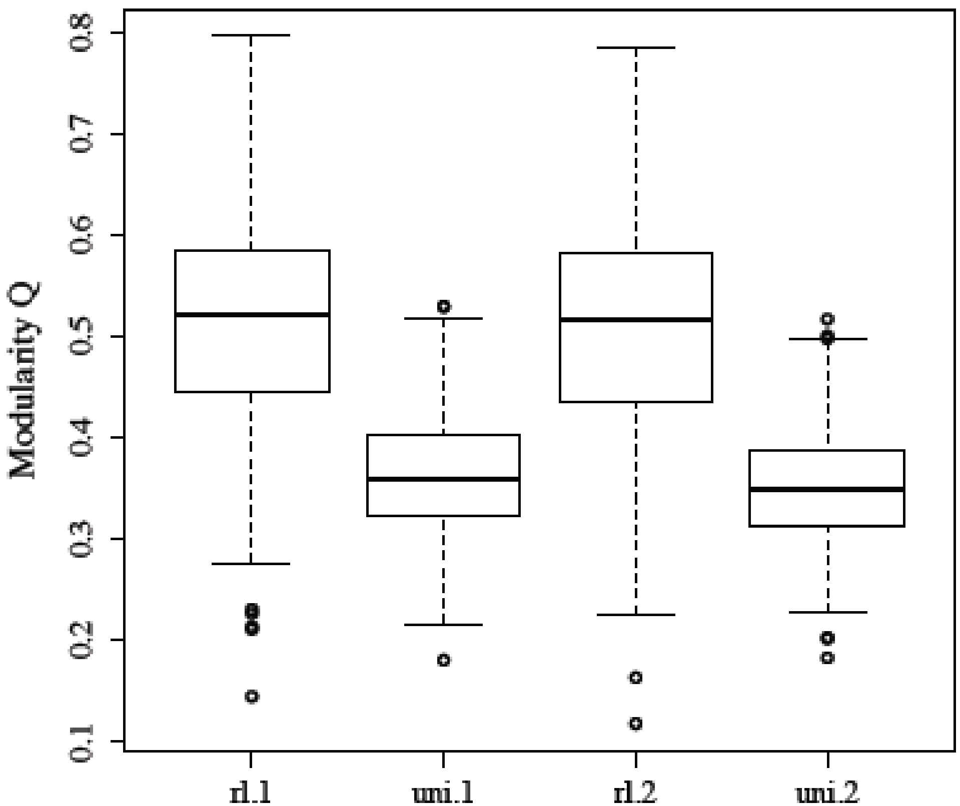



Networks are too dense!

Filtering process to keep only the more likely transitions

- Establish a threshold *T* for the weights
- Suppress all edges with $W_{ij}$ smaller than the value marking the *T*-quantile in the weights distribution

# Results: Modularity Score $Q$




- Two classes well separated
- *Real-like* instances have significantly more clustering
- The community detection algorithm has no influence

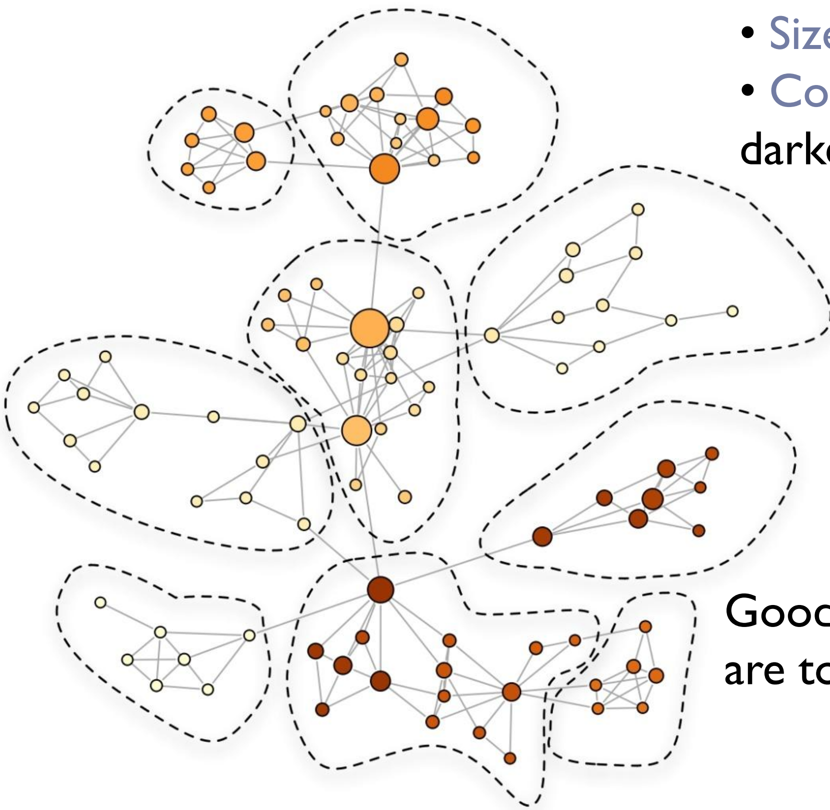
$Q$ with respect to class of problem (r = *real-like*, uni = *random*), and the two community detection algorithms

# Summary and Conclusions

**Nodes**

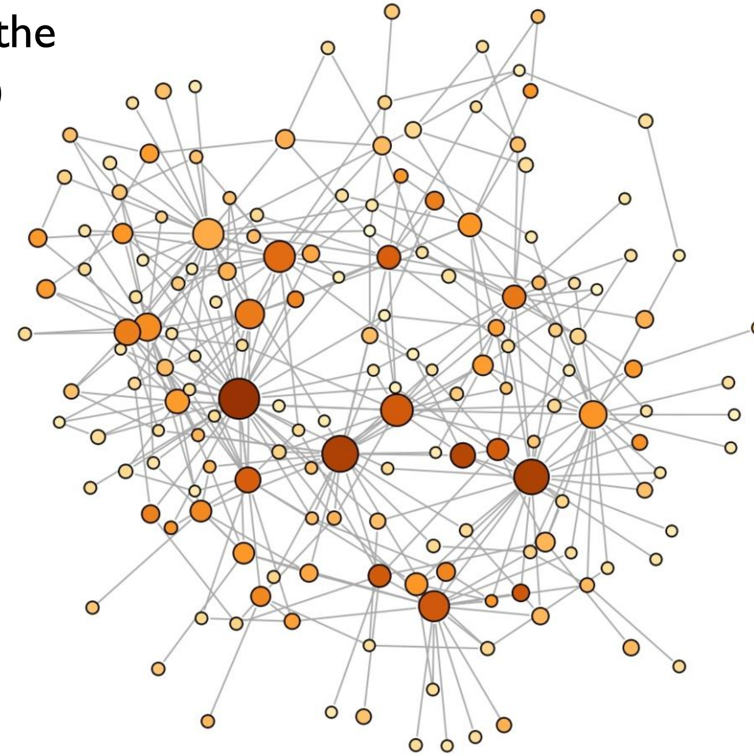
- Size : size of basins
- Colour: fitness (the darker the better)

## Implications for search?



Good solutions are together!

*Real-like* instance ($Q$ = 0.79)



*Random* instance ($Q$ = 0.53)

F. Daolio, M. Tomassini, S. Verel, G. Ochoa (2011) Communities of Minima in Local Optima Networks of Combinatorial Spaces, *Physica A* (to appear).